The derivative of a fuction is $f'(x) = (x-1)^2(x+3)$. Find the value of x at each point where f has a:

a. Local maximum
b. Local minimum
c. Point of inflection

Solution:

$$f'(x) = (x-1)^2(x+3) = 0 \to x_1 = 3, x_{2,3} = 1$$

$$f''(x) = 2(x-1)(x+3) + (x-1)^2 = 2(x-1)(x+2)$$

f has not local maximum, but it has local minimum at $x = -3$.

Point of inflection are $f''(x) = 0 \to x_1 = -2, x_2 = 1$

Answer:

a. This function has not local maximum
b. Local minimum at $x = -3$
c. Point of inflection are $x_1 = -2, x_2 = 1$.