Conditions

let x be a nonempty set and let f: X->R have bounded range in R. if a element R , show that
sup(a+f(x): x element X)= a + sup (f(x): x element X)
and
inf(a+f(x): x element X)= a + inf (f(x): x element X)

Solution

$$X \neq \emptyset, f: X \rightarrow R, a \epsilon R$$

Consider

s $= \sup(\{a + f(x)\}) \leftrightarrow s \epsilon S_x \ \forall y \epsilon S_x: s \leq y, where\ S_x = \{y \epsilon M \,|\, \forall x \epsilon X: x \leq y\}$

$\forall x \epsilon X, a + f(x) \leq s \leq y$

$\forall x \epsilon X, f(x) \leq s - a$

$s - a\ is\ a\ supremum\ for\ \{f(x), x \epsilon X\}$

Consider

$\boldsymbol{a} + \mathbf{sup}(\{\boldsymbol{f}(\boldsymbol{x})\}) = s - a + a = s, which\ is\ \mathbf{sup}(\{\boldsymbol{a} + \boldsymbol{f}(\boldsymbol{x})\})$

The same way is how to proof that

$\inf(\{a + f(x)\}) = a + \inf(\{f(x)\})$

$\inf(\{f(x)\} = i, \forall x \epsilon X\ x \geq i, \forall x \epsilon X\ x + a \geq i + a$

So, $\inf(\{a + f(x)\} = i + a = a + \inf(\{f(x)\})$