

Conditions

look at matrix

$$Z = \begin{pmatrix} 1 & -1 \\ 2 & 4 \end{pmatrix}$$

write matrix Z in its Jordan form ? Please show your work

Solution

In linear algebra, a **Jordan normal form** (often called **Jordan canonical form**)^[1] of a linear operator on a finite-dimensional vector space is an upper triangular matrix of a particular form called a Jordan matrix, representing the operator on some basis. The form is characterized by the condition that any non-diagonal entries that are non-zero must be equal to 1, be immediately above the main diagonal (on the superdiagonal), and have identical diagonal entries to the left and below them. If the vector space is over a field K , then a basis on which the matrix has the required form exists if and only if all eigenvalues of M lie in K , or equivalently if the characteristic polynomial of the operator splits into linear factors over K . This condition is always satisfied if K is the field of complex numbers. The diagonal entries of the normal form are the eigenvalues of the operator, with the number of times each one occurs being given by its algebraic multiplicity.

If the operator is originally given by a square matrix M , then its Jordan normal form is also called the Jordan normal form of M . Any square matrix has a Jordan normal form if the field of coefficients is extended to one containing all the eigenvalues of the matrix. In spite of its name, the normal form for a given M is not entirely unique, as it is a block diagonal matrix formed of Jordan blocks, the order of which is not fixed; it is conventional to group blocks for the same eigenvalue together, but no ordering is imposed among the eigenvalues, nor among the blocks for a given eigenvalue, although the latter could for instance be ordered by weakly decreasing size. The Jordan–Chevalley decomposition is particularly simple on a basis on which the operator takes its Jordan normal form. The diagonal form for diagonalizable matrices, for instance normal matrices, is a special case of the Jordan normal form.

The Jordan normal form is named after Camille Jordan.

For our case:

$$M = SJS^{-1}$$

$$M = \begin{pmatrix} 1 & -1 \\ 2 & 4 \end{pmatrix}$$

The eigenvalues of M :

$$\begin{vmatrix} 1 - \lambda & -1 \\ 2 & 4 - \lambda \end{vmatrix} = (1 - \lambda)(4 - \lambda) + 2 = 6 - 5\lambda + \lambda^2$$

$$\lambda_1 = 2, \lambda_2 = 3$$

$$J = \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$$

Answer: Jordan form

$$J = \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$$