Conditions

look at vectors

$$u=\begin{pmatrix}1\\0\\1\end{pmatrix} \quad v=\begin{pmatrix}1\\1\\0\end{pmatrix} \quad w=\begin{pmatrix}0\\1\\-1\end{pmatrix}$$

what is the dimension of the space spanned by these three vectors ?

is the vector $\begin{pmatrix}1\\-1\\4\end{pmatrix}$ in tyhe span of vectors u , v , and w ?

Please explain

Solution

In mathematics, the **dimension** of a vector space *V* is the cardinality (i.e. the number of vectors) of a basis of *V.* In linear algebra, a **basis** is a set of linearly independent vectors that, in a linear combination, can represent every vector in a given vector space or free module, or, more simply put, which define a "coordinate system" (as long as the basis is given a definite order). In more general terms, a basis is a linearly independent spanning set.

Given a basis of a vector space, every element of the vector space can be expressed uniquely as a finite linear combination of basis vectors. Every vector space has a basis, and all bases of a vector space have the same number of elements, called the dimension of the vector space.

Let's check, if u,v,w is linear independent.

$$\begin{pmatrix}1 & 0 & 1\\1 & 1 & 0\\0 & 1 & -1\end{pmatrix}$$

As we see, the 1st element of w is 0, the 2nd element of u is 0 and the 3rd element of v is 0. This means that there is no exist a linear combination, which could transform one of these vectors to another. That means that u,v,w is a basis, and the dimension space, spanned by these 3 vectors is:

$$(x,y,z)=c_1(1,0,1)+c_2(1,1,0)+c_3(0,1,-1)=(c_1+c_2,c_2+c_3,c_1-c_3)$$

Let's check, if the vector $(1,-1,4)$ is in the span of u,v,w?

For this let's find c_1, c_2, c_3 for which the previous equation is correct.

$$(1,-1,4)=(c_1+c_2,c_2+c_3,c_1-c_3)$$

$$\begin{cases}c_1+c_2=1\\c_2+c_3=-1\\c_1-c_3=4\end{cases}$$

$$c_1=1-c_2$$

$$c_2=-1-c_3$$

$$1+1+c_3-c_3=4$$

No solution exist.

That means, that the vector $(1, -1, 4)$ is **not from a subspace, spanned by u,v,w.**