Conditions

Using the definition of the limit at infinity verify that lim x-->infinity cos^2(x)/2x^2 = 0

Solution

Consider the function f(x) with a limit, equal to F, when x→infinity:

$\forall \varepsilon > 0\ \exists \delta = \delta(\varepsilon) > 0\ \forall x{:}\ |x| > \delta\ |f(x) - F| < \varepsilon$

In our case F = 0. Let's verify, that the limit is equal to F.

For this let's fix $\varepsilon > 0$

$$|f(x) - F| = \left|\frac{cos^2(x)}{2x^2} - 0\right| = \left|\frac{cos^2(x)}{2x^2}\right| \le \left|\frac{1}{2x^2}\right| < \left|\frac{1}{2\delta^2}\right| = \frac{1}{2\delta^2} < \varepsilon$$

$$\delta = \sqrt{\frac{1}{2\varepsilon}}$$

We've got, that

$$\forall \varepsilon > 0\ \exists \delta = \delta(\varepsilon) = \sqrt{\frac{1}{2\varepsilon}} > 0\ \forall x{:}\ |x| > \delta\ \left|\frac{cos^2(x)}{2x^2}\right| < \varepsilon$$

Q.E.D.