

6 friends (Andy, Bandy, Candy, Dandy, Endy and Fandy) are out to dinner. They will be seated in a circular table (with 6 seats). Andy and Bandy want to sit next to each other to talk about the Addition Principle, Bandy and Candy want to sit next to each other to talk about the Principle of Inclusion and Exclusion. How many ways are there to seat them? Clarification: Rotations are counted as the same seating arrangements, reflections are counted as different seating arrangements.

Solution:

For the first seat, we have a choice of any of the 6 friends. After seating the first person, for the second seat, we have a choice of any of the remaining 5 friends. After seating the second person, for the third seat, we have a choice of any of the remaining 4 friends. After seating the third person, for the fourth seat, we have a choice of any of the remaining 3 friends. After seating the fourth person, for the fifth seat, we have a choice of any of the remaining 2 friends. After seating the fifth person, for the sixth seat, we have a choice of only 1 of the remaining friends. Hence, by the Rule of Product, there are $6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$ ways to seat these 6 people.

More generally, this problem is known as a Permutation. There are $n! = n \times (n - 1) \times (n - 2) \times \dots \times 1$ ways to seat n people in a row.