Conditions

Using the definition of the limit at infinity verify that lim x-->infinity cos^2(x)/2x^2 = 0

Solution

Definition. The function f(x) has a limit, equal to F, when x→infinity, if:

$\forall \varepsilon > 0\ \exists \delta = \delta(\varepsilon) > 0\ \forall x: |x| > \delta\ |f(x) - F| < \varepsilon$

Fix $\varepsilon > 0$

Consider $|f(x) - F|$ for our case:

$$\left|\frac{cos^2(x)}{2x^2} - 0\right| = \left|\frac{cos^2(x)}{2x^2}\right| \leq \left|\frac{1}{2x^2}\right| < \left|\frac{1}{2\delta^2}\right| = \frac{1}{2\delta^2} < \varepsilon$$

Here $\delta = \sqrt{\frac{1}{2\varepsilon}}$

So, for each ε we found $\delta = \delta(\varepsilon)$, for which $\left|\frac{cos^2(x)}{2x^2} - 0\right| < \varepsilon$

Q.E.D.