Using the epsilon-delta definition verify that $\lim_{x\to 3}(x^2 - x) = 6$

Solution: Begin by letting $\varepsilon > 0$ be given. Find $\delta > 0$ (which depends on ε) so that if $0 < |x - 3| < \delta$, then $|f(x) - 6| < \varepsilon$. Begin with $|f(x) - 6| < \varepsilon$ and solve for $|x - 3|$. Then

$$|f(x) - 6| < \varepsilon \rightarrow |x^2 - x - 6| < \varepsilon$$

$$|x - 3||x + 2| < \varepsilon$$

We will now replace the term $|x + 2|$ with an appropriate constant and keep the term $|x - 3|$, since this this is the term we wish to solver for. To do this we will arbitrarily assume that $\delta \le 1$. Then $|x - 3| < \delta \le 1$ implies that $-1 < x - 3 < 1$ and $2 < x < 4$ so that $7 < |x + 2| < 9$. It follows that

$$|x - 3||x + 2| < |x - 3|(9) < \varepsilon \rightarrow |x - 3| < \frac{\varepsilon}{9}.$$

Now choose $\delta = \min\{4, \frac{\varepsilon}{9}\}$. Thus if $0 < |x - 3| < \delta$, t follows that $|f(x) - 6| < \varepsilon$. This completes the proof.