

Question

With induction:

1) We have: $n = 1$: $2^{3 \cdot 1} - 1 = 8 - 1 = 7$ divide by 7.

2) Let take that when $n = k$: $2^{3k} - 1$ divide by 7 $\Rightarrow 2^{3k} - 1 = 7m$.

3) And when:

$$\begin{aligned} n = k + 1: 2^{3(k+1)} - 1 &= 8 \cdot 2^{3k} - 1 = 8 \cdot 2^{3k} - 8 + 7 = 8 \cdot (2^{3k} - 1) + 7 = \\ &= 8 \cdot 7m + 7 = 7 \cdot (8m + 1) \Rightarrow \text{divide by 7.} \end{aligned}$$

So, we proved that 7 divides $2^{3n} - 1$.

Answer: Proved.