Question

a) Function $f(x)=\frac{x^3}{e^{(1-x^2)}}$. Domain of the function is: $x\in R$.

$$f'(x)=\left(\frac{x^3}{e^{(1-x^2)}}\right)'=\frac{3x^2\cdot e^{(1-x^2)}-x^3\cdot e^{(1-x^2)}\cdot(-2x)}{e^{2\cdot(1-x^2)}}=\frac{3x^2+2x^4}{e^{(1-x^2)}}.$$

$$f''(x)=\left(\frac{3x^2+2x^4}{e^{(1-x^2)}}\right)'=\frac{(6x+8x^3)\cdot e^{(1-x^2)}-(3x^2+2x^4)\cdot e^{(1-x^2)}\cdot(-2x)}{e^{2\cdot(1-x^2)}}=$$

$$=\frac{4x^5+14x^3+6x}{e^{(1-x^2)}}=\frac{2x\cdot(2x^4+7x^2+3)}{e^{(1-x^2)}}.$$

$$f'(x)=\frac{3x^2+2x^4}{e^{(1-x^2)}}=0\Rightarrow x=0.$$

$f''(0)=0\Rightarrow$ it's a inflection point: $(0,0)$.

$$f''(0)=\frac{2x\cdot(2x^4+7x^2+3)}{e^{(1-x^2)}}=0\Rightarrow\begin{cases}x=0\\2x^4+7x^2+3=0\end{cases}\Rightarrow x=0 \text{ - inflection point.}$$

So, there is no minimum and maximum points and there is one inflection point: $(0,0)$.

b) We have:

Vertical asymptote: *none*.

$$\begin{cases}\lim\limits_{x\to-\infty}\left(\frac{x^3}{e^{(1-x^2)}}\right)=-\infty\\ \lim\limits_{x\to+\infty}\left(\frac{x^3}{e^{(1-x^2)}}\right)=\infty\end{cases}\Rightarrow \text{there is no any horizontal assymptotes.}$$

c) There x- intercept: $\frac{x^3}{e^{(1-x^2)}}=0\Rightarrow x=0\Rightarrow$ x-intercept: $(0,0)$. And y-intercept:

$$f(0)=\frac{0^3}{e^{(1-0^2)}}=0\Rightarrow \text{ y-intercept: } (0,0).$$