Question

a) Function $f\left(x\right)=\frac{x^3}{e^{\left(1-x^2\right)}}$. Domain of the function is: $x\in R$.

$$f'\left(x\right)=\left(\frac{x^3}{e^{\left(1-x^2\right)}}\right)'=\frac{3x^2\cdot e^{\left(1-x^2\right)}-x^3\cdot e^{\left(1-x^2\right)}\cdot\left(-2x\right)}{e^{2\cdot\left(1-x^2\right)}}=\frac{3x^2+2x^4}{e^{\left(1-x^2\right)}}.$$

$$f''\left(x\right)=\left(\frac{3x^2+2x^4}{e^{\left(1-x^2\right)}}\right)'=\frac{\left(6x+8x^3\right)\cdot e^{\left(1-x^2\right)}-\left(3x^2+2x^4\right)\cdot e^{\left(1-x^2\right)}\cdot\left(-2x\right)}{e^{2\cdot\left(1-x^2\right)}}=$$

$$=\frac{4x^5+14x^3+6x}{e^{\left(1-x^2\right)}}=\frac{2x\cdot\left(2x^4+7x^2+3\right)}{e^{\left(1-x^2\right)}}.$$

b) We have:

$$\begin{cases} f\left(x\right)=\frac{x^3}{e^{\left(1-x^2\right)}} \\ f\left(-x\right)=\frac{\left(-x\right)^3}{e^{\left(1-\left(-x\right)^2\right)}}=\frac{-x^3}{e^{\left(1-x^2\right)}}=-\frac{x^3}{e^{\left(1-x^2\right)}}=-f\left(x\right) \end{cases}\Rightarrow \text{function is odd.}$$

c) Critical points:

$$f'\left(x\right)=\frac{3x^2+2x^4}{e^{\left(1-x^2\right)}}=0\Rightarrow x=0.$$

$$f''\left(0\right)=0\Rightarrow \text{it's a inflection point: }\left(0,0\right).$$

The critical point (it’s an inflection point): $\left(0,0\right)$.