

Question

a) Function $f(x) = \frac{(3x-2)}{(3x+1)}$. Domain of the function is: $x \in \left(-\infty, -\frac{1}{3}\right) \cup \left(-\frac{1}{3}, \infty\right)$.

$$f'(x) = \left(\frac{3x-2}{3x+1}\right)' = \frac{3 \cdot (3x+1) - 3 \cdot (3x-2)}{(3x+1)^2} = \frac{9x+3-9x+6}{(3x+1)^2} = \frac{9}{(3x+1)^2}$$

$$f''(x) = \left(\frac{9}{(3x+1)^2}\right)' = \frac{-2 \cdot 3 \cdot 9 \cdot (3x+1)}{(3x+1)^4} = -\frac{54}{(3x+1)^3}$$

So, we can say that there is no any critical points which are in the domain of this function: there is no maximum, minimum or inflection points.

b) We have:

Vertical asymptote: $x = -\frac{1}{3}$.

$$\begin{cases} \lim_{x \rightarrow -\infty} \left(\frac{3x-2}{3x+1}\right) = \lim_{x \rightarrow -\infty} \left(\frac{3-2/x}{3+1/x}\right) = 1 \\ \lim_{x \rightarrow \infty} \left(\frac{3x-2}{3x+1}\right) = \lim_{x \rightarrow \infty} \left(\frac{3-2/x}{3+1/x}\right) = 1 \end{cases} \Rightarrow \text{there is a horizontal asymptote } y = 1.$$

c) There x- intercept: $\frac{3x-2}{3x+1} = 0 \Rightarrow x = \frac{2}{3} \Rightarrow$ x-intercept $\left(\frac{2}{3}, 0\right)$. And y-intercept:

$$f(0) = \frac{0-2}{0+1} = -2 \Rightarrow \text{y-intercept: } (0, -2).$$