

Question

a) Function $f(x) = \frac{(3x-2)}{(3x+1)}$. Domain of the function is: $x \in \left(-\infty, -\frac{1}{3}\right) \cup \left(-\frac{1}{3}, \infty\right)$.

$$f'(x) = \left(\frac{3x-2}{3x+1}\right)' = \frac{3 \cdot (3x+1) - 3 \cdot (3x-2)}{(3x+1)^2} = \frac{9x+3-9x+6}{(3x+1)^2} = \frac{9}{(3x+1)^2}$$

$$f''(x) = \left(\frac{9}{(3x+1)^2}\right)' = \frac{-2 \cdot 3 \cdot 9 \cdot (3x+1)}{(3x+1)^4} = -\frac{54}{(3x+1)^3}$$

There is one candidate in inflection point: $x = -\frac{1}{3}$. But this point doesn't include in domain of the function. So, there is no any inflection points.

b) Function increasing when:

$$f'(x) = \frac{9}{(3x+1)^2} > 0 \Rightarrow x \in \left(-\infty, -\frac{1}{3}\right) \cup \left(-\frac{1}{3}, \infty\right).$$

Function decreasing when:

$$f'(x) = \frac{9}{(3x+1)^2} < 0 \Rightarrow x \in \emptyset.$$

c) Function concave up when:

$$f''(x) = -\frac{54}{(3x+1)^3} < 0 \Rightarrow \frac{54}{(3x+1)^3} > 0 \Rightarrow x \in \left(-\frac{1}{3}, \infty\right).$$

Function concave down when:

$$f''(x) = -\frac{54}{(3x+1)^3} > 0 \Rightarrow \frac{54}{(3x+1)^3} < 0 \Rightarrow x \in \left(-\infty, -\frac{1}{3}\right).$$