Question

The function $f(x)=\frac{x+1}{x^2-1}$:

$$f(x)=\frac{x+1}{x^2-1}=\frac{1}{x-1}$$

$$f'(x)=-\frac{1}{(x-1)^2}$$

$$f''(x)=\frac{2}{(x-1)^3}$$

Domain: $x\in(-\infty,1)\cup(1,\infty)$.

a) We have:

$f''(x)=\frac{2}{(x-1)^3}\Rightarrow$ there is one candidate for inflection point – it's $x=1$. But this point is not in domain of this function. So, there is no inflection points.

b) Function increasing when: $f'(x)=-\frac{1}{(x-1)^2}>0\Rightarrow\frac{1}{(x-1)^2}<0\Rightarrow x\in\varnothing$. Function decreasing when: $f'(x)=-\frac{1}{(x-1)^2}<0\Rightarrow\frac{1}{(x-1)^2}>0\Rightarrow x\in(-\infty,1)\cup(1,\infty)$.

c) Function concave up when: $f''(x)=\frac{2}{(x-1)^3}<0\Rightarrow x\in(-\infty,1)$. Function concave down when: $f''(x)=\frac{2}{(x-1)^3}>0\Rightarrow x\in(1,\infty)$.