Conditions

Show that if lim a_n = -infinity, then lim 1/a_n = 0

Solution

The limit of $\{a_n\} = -\infty$ is by the definition means, that:

$\forall \varepsilon > 0 \ \exists N = N(\varepsilon) \ \forall n \geq N \ a_n < -\varepsilon$

Let's consider the $lim \frac{1}{a_n}$. If we want to prove that the limit is equal to 0, then we must prove the following:

$$\forall \varepsilon > 0 \ \exists N = N(\varepsilon) \ \forall n \geq N \ \left|\frac{1}{a_n}\right| < \varepsilon$$

Fix $\varepsilon > 0$, $\exists N = N(\varepsilon) \ \forall n \geq N \ a_n < -\varepsilon$.

If $a_n < -\varepsilon$ then for big numbers of n, $-\varepsilon < \frac{1}{a_n}$, but as $\frac{1}{a_n} < 0$ and $\varepsilon > 0$ then:

$$-\varepsilon < \frac{1}{a_n} < \varepsilon$$

Which is definitely means $\left|\frac{1}{a_n}\right| < \varepsilon$

Prove is done.