$$\begin{cases} x + y - 2z = 0 \\ x - 2y + z = 0 \\ x - y - z = -1 \end{cases}$$

Solution

$$AX = B$$

$$A = \begin{pmatrix} 1 & 1 & -2 \\ 1 & -2 & 1 \\ 1 & -1 & -1 \end{pmatrix} \qquad X = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \qquad B = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}$$

$$X = A^{-1}B$$

Find the inverse matrix A^{-1}

$$det\ A = \begin{vmatrix} 1 & 1 & -2 \\ 1 & -2 & 1 \\ 1 & -1 & -1 \end{vmatrix} = 1(2+1) - 1(-1-2) + 1(1-4) = 3 + 3 - 3 = 3$$

$$A^{-1} = \frac{1}{det\ A} \begin{pmatrix} 3 & 3 & -3 \\ 2 & 1 & -3 \\ 1 & 2 & -3 \end{pmatrix}$$

$$A^{-1} = \begin{pmatrix} 1 & 1 & -1 \\ \dfrac{2}{3} & \dfrac{1}{3} & -1 \\ \dfrac{1}{3} & \dfrac{2}{3} & -1 \end{pmatrix}$$

$$X = \begin{pmatrix} 1 & 1 & -1 \\ \dfrac{2}{3} & \dfrac{1}{3} & -1 \\ \dfrac{1}{3} & \dfrac{2}{3} & -1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}$$

$$X = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$x = 1, \qquad y = 1, \qquad z = 1$$

Answer: $x = 1,\ y = 1, \qquad z = 1$