Conditions

Given three points A=(2, -3, 4), B=(0, 1, 2), and C=(-1, 2, 0) in a 3-D coordinate systems. Find the area of the triangle with A, B, and C as vertices.

Solution

It's known, that the area of triangle in R^3could be found by using the following formula:

$$S = \sqrt{S_x^2 + S_y^2 + S_z^2},$$

where

$$S_x = \frac{1}{2}\begin{vmatrix} 1 & y_A & z_A \\ 1 & y_B & z_B \\ 1 & y_C & z_C \end{vmatrix} = \frac{1}{2}\begin{vmatrix} 1 & -3 & 4 \\ 1 & 1 & 2 \\ 1 & 2 & 0 \end{vmatrix}$$

$$S_y = \frac{1}{2}\begin{vmatrix} x_A & 1 & z_A \\ x_B & 1 & z_B \\ x_C & 1 & z_C \end{vmatrix} = \frac{1}{2}\begin{vmatrix} 2 & 1 & 4 \\ 0 & 1 & 2 \\ -1 & 1 & 0 \end{vmatrix}$$

$$S_z = \frac{1}{2}\begin{vmatrix} x_A & y_A & 1 \\ x_B & y_B & 1 \\ x_C & y_C & 1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} 2 & -3 & 1 \\ 0 & 1 & 1 \\ -1 & 2 & 1 \end{vmatrix}$$

Let's calculate it

$$S_x = -3$$

$$S_y = -1$$

$$S_z = 1$$

$$S = \sqrt{9 + 1 + 1} = \sqrt{\mathbf{11}}$$