$$f(x) = x^a (1-x)^b,\ 0 \le x \le 1$$

$$f'(x) = x^{a-1}\left(-(1-x)^{b-1}\right)\left(a(x-1) + bx\right) = 0 \Rightarrow$$

$$\Rightarrow x = 0, x = 1, x = \frac{a}{a+b}.$$

$$f(x=0) = 0,$$

$$f(x=1) = 0,$$

$$f\left(x = \frac{a}{a+b}\right) = \left(\frac{a}{a+b}\right)^a \left(\frac{b}{a+b}\right)^b \Rightarrow$$

$$\Rightarrow f_{max} = \left(\frac{a}{a+b}\right)^a \left(\frac{b}{a+b}\right)^b \text{ at } x = \frac{a}{a+b}.$$

<u>Answer:</u> $f_{max} = \left(\frac{a}{a+b}\right)^a \left(\frac{b}{a+b}\right)^b$ at $x = \frac{a}{a+b}.$