Question

Function $f\left(x\right)=\dfrac{x^2-4}{x^2+4}$ on the interval $\left[-5,5\right]$.

$$f^{'}\left(x\right)=\left(\frac{x^2-4}{x^2+4}\right)^{'}=\frac{2x\left(x^2+4\right)-2x\left(x^2-4\right)}{\left(x^2+4\right)^2}=\frac{16x}{\left(x^2+4\right)^2}$$

Critical points: $x=0:f\left(0\right)=-1$.

As we see $f^{'}\left(x\right)>0$ when $x>0$ and $f^{'}\left(x\right)<0$ when $x<0$. So, it's minimum.

We have:

$$\begin{cases} x=-5:f\left(-5\right)=\frac{21}{29}; \\ x=0:f\left(0\right)=-1; \\ x=5:f\left(0\right)=\frac{21}{29}; \end{cases} \Rightarrow \begin{cases} \text{absolute maximum on the interval [-5,5] equals } \frac{21}{29}\text{, when x = -5 and x = 5} \\ \text{absolute minimum on the interval [-5,5] equals -1, when x = 0} \end{cases}$$

<u>Answer:</u> $\begin{cases} \text{absolute maximum on the interval [-5,5] equals } \frac{21}{29}\text{, when x = -5 and x = 5} \\ \text{absolute minimum on the interval [-5,5] equals -1, when x = 0} \end{cases}$.