Question

Function $f(x) = x^3 - 6x^2 + 9x + 6$ on the interval $[-1, 4]$.

$f'(x) = 3x^2 - 12x + 9$

Critical points:

$f'(x) = 3x^2 - 12x + 9 = 0 \Rightarrow$

$\Rightarrow x = \frac{12 \pm \sqrt{144 - 108}}{6} = \frac{12 \pm 6}{6} = 2 \pm 1 = \{1,3\}.$

We have:

$$\begin{cases} x = -1 : f(-1) = -1 - 6 - 9 + 6 = -10; \\ x = 1 : f(1) = 1 - 6 + 9 + 6 = 10; \\ x = 3 : f(3) = 27 - 54 + 27 + 6 = 6; \\ x = 4 : f(4) = 64 - 96 + 36 + 6 = 10; \end{cases} \Rightarrow \begin{cases} \text{absolute maximum on the interval [-1,4] equals 10, when x = 1 and x = 4} \\ \text{absolute minimum on the interval [-1,4] equals - 10, when x = -1} \end{cases}$$

Answer: $\begin{cases} \text{absolute maximum on the interval [-1,4] equals 10, when x = 1 and x = 4} \\ \text{absolute minimum on the interval [-1,4] equals - 10, when x = -1} \end{cases}$.