Let A be a commutative discrete valuation ring with a uniformizer π(*that is nonzero*) and quotient field K. Consider the ring $R = \begin{pmatrix} A & K \\ 0 & K \end{pmatrix}$, which is right noetherian (but not left noetherian). It is easy to check that

$$J := \begin{pmatrix} \pi A & K \\ 0 & 0 \end{pmatrix}$$

is an ideal of R, and that $R/J \sim (A/\pi A) \times K$. Since the latter is a semisimple ring, we have rad $R \subseteq J$. On the other hand, $1 + J$ consists of matrices of the form

$$\begin{pmatrix} 1+\pi a & b \\ 0 & 1 \end{pmatrix} (a \in A, b \in K),$$

which are clearly units of R. Therefore, $J \subseteq$ rad R. We have now$J$ = rad R, from which it is easy to see that

$$(\text{rad } R)^n = \begin{pmatrix} \pi^n A & K \\ 0 & 0 \end{pmatrix},$$

for any $n \geq 1$. It follows that $\bigcap_{n \geq 1} (rad\ R)^n = \begin{pmatrix} 0 & K \\ 0 & 0 \end{pmatrix} \neq 0$.