To construct an artinian ring R for which the two socles differ, take $R = \begin{pmatrix} Q & Q \\ 0 & Q \end{pmatrix}$. For this 3-dimensional Q-algebra, we have rad $R = \begin{pmatrix} 0 & Q \\ 0 & 0 \end{pmatrix}$, which has right annihilator $\begin{pmatrix} Q & Q \\ 0 & 0 \end{pmatrix}$ and left annihilator $\begin{pmatrix} 0 & Q \\ 0 & Q \end{pmatrix}$, so soc($RR$) <> soc($RR$).