

If we will take that $a \geq b \geq c$, then we will have:

$$\log_{b^m c^n} a + \log_{c^m a^n} b + \log_{a^m b^n} c \geq \frac{3}{(m+n)}$$

$$\begin{aligned} \log_{b^m c^n} a + \log_{c^m a^n} b + \log_{a^m b^n} c &= \\ &= \frac{1}{\log_a b^m c^n} + \frac{1}{\log_b c^m a^n} + \frac{1}{\log_c a^m b^n} = \\ &= \frac{1}{m \log_a b + n \log_a c} + \frac{1}{m \log_b c + n \log_b a} + \frac{1}{m \log_c a + n \log_c b} \geq \\ &\geq \frac{1}{m+n} + \frac{1}{m+n} + \frac{1}{m+n} = \frac{3}{m+n}. \end{aligned}$$

Proved.

Answer: Proved.