

Let $J = \text{rad}(R)$, $J' = \text{rad}(S)$, and $I = \begin{pmatrix} J & M \\ 0 & J' \end{pmatrix}$. It is routine to check that I is an ideal of T , with $T/I \sim R/J \times S/J'$. The latter ring is J -semisimple, so we have $\text{rad}(T) \subseteq I$. This will be an equality, as asserted, if we can show that $1 + I \subseteq U(T)$. Now any element of $1 + I$ has the form $\begin{pmatrix} u & m \\ 0 & v \end{pmatrix}$, where $m \in M$, $u \in U(R)$, and $v \in U(S)$. This is indeed in $U(T)$, since it has inverse $\begin{pmatrix} u^{-1} & -u^{-1}mv^{-1} \\ 0 & v^{-1} \end{pmatrix}$.