Conditions

1. Consider the function f(x)=x^3-2 using Newton's method. Take x0=1.5 for the starting value.

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For each method, present the results in the form of table:

Column1:n (step)

Column2:xn (approximation)

Column3: ( ) n f x

Column4:| xn - xn-1 | (error)

Solution

In numerical analysis, **Newton's method** (also known as the **Newton–Raphson method**), named after Isaac Newton and Joseph Raphson, is a method for finding successively better approximations to the roots (or zeroes) of a real-valued function.

$$x : f(x) = 0\,.$$

The algorithm is first in the class of Householder's methods, succeeded by Halley's method. The method can also be extended to complex functions and to systems of equations.

The Newton-Raphson method in one variable is implemented as follows:

Given a function *f* defined over the reals *x*, and its derivative *f* ', we begin with a first guess x_0 for a root of the function *f*. Provided the function satisfies all the assumptions made in the derivation of the formula, a better approximation x_1 is

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}\,.$$

Geometrically, (x_1, 0) is the intersection with the *x*-axis of a line tangent to *f* at (x_0, $f(x_0)$).

The process is repeated as

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

until a sufficiently accurate value is reached.

For our example:

n	Xn	f(Xn)	ABS(Xn+1 - Xn)
0	1,5000000000000	1,3750000000000	
1	1,2962962962963	0,1782756693593	0,2037037037037
2	1,2609322247418	0,0048192857926	0,0353640715545
3	1,2599218605659	0,0000038605827	0,0010103641758
4	1,2599210498954	0,0000000000025	0,0000008106705
5	1,2599210498949	0,0000000000000	0,0000000000005
6	1,2599210498949	0,0000000000000	0,0000000000000

Answer:

As we see, after 6th iteration the root has been found, and it’s approximately equal to:

$x = 1.2599210498949$