Question 1. *Show that if ab is left quasi-regular element of ring, then so is ba.*

Solution. Suppose ab is quasi-regular, so there is c such that $0 = c \circ ab = ab \circ c$. Consider $d = b(c-1)a$. Note that $d \circ ba = b(c-1)a + ba - b(c-1)aba = bca - ba + ba - bcaba + baba = b(c - cab + ab)a = b(c \circ ab)a = b \cdot 0 \cdot a = 0$. And similarly $ba \circ d = ba + b(c-1)a - bab(c-1)a = ba + bca - ba - babca + baba = b(c - abc + ab)a = b(ab \circ c)a = b \cdot 0 \cdot a = 0$. Thus, ba is quasi-regular.

$\square$

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