

A particle is moving along the curve $y = \sqrt{x}$. As the particle passes through the point (4,2), its x-coordinate increases at a rate of 3cm/s. How fast is the distance from the particle to the origin changing at this instant?

..... 3,02 cm/s

Solution

The key is to construct a right triangle to determine the path of the object.

$$z^2 = x^2 + y^2$$

When taken its derivative: $z \frac{dz}{dt} = x \frac{dx}{dt} + y \frac{dy}{dt}$

1. Using the point (4,2) solve for z (first equation)

$$z^2 = 4^2 + 2^2 = 20 \gg z = 4,47$$

2. Using $y = \sqrt{x}$, find dy/dt noting that $x = 4$ as $dx/dt = 3$ cm/s

$$\frac{dy}{dt} = y' \times \frac{dx}{dt} = \frac{0.5}{4^{0.5}} \times 3 = 0.75$$

3. From these, solve for $dz/dt =$ rate of distance from particle to origin.
first take the derivative of the function

$$4,47 \frac{dz}{dt} = 4 \times 3 + 2 \times 0.75 = 13.5 \gg \frac{dz}{dt} = \frac{13.5}{4,47} = 3,02 \frac{\text{cm}}{\text{s}}$$