Brain weight B as a function of body weight W in fish has been modeled by the power function B=.007W^(2/3), where B and W are measured in grams. A model for body weight as a function of body length L (measured in cm) is W=.12L^(2.53). If, over 10 million years, the average length of a certain species of fish evolved from 15cm to 20cm at a constant rate, how fast was the species' brain growing when the average length was 18cm? Round your answer to the nearest hundredth.

=10.4...nanograms/yr

Solution

The length is growing at a constant rate (this means the rate of change of L over time, dL/dt, is constant) from 15 to 20 cms over a period of 10^7 years or

$\frac{dL}{dt} = \frac{5}{10^7} = 5 * 10^{-7}$

$$\frac{dW}{dt} = 0.12 * 2.53L^{1.53} \frac{dL}{dt}$$

... taking the derivative using the power rule.

$$\frac{dW}{dt} = 0.3036 * (18)^{1.53} * 5 * 10^{-7} = 1.26 * 10^{-5}$$

$$W = 0.12 * 18^{2.53} = 1.8 * 10^2$$

Now, $\frac{dB}{dt} = 0.007 * \left(\frac{2}{3}\right) W^{-\frac{1}{3}} \frac{dW}{dt}$

Substitute here what you've computed for W and dW/dt to find the desired dB/dt.

$$\frac{dB}{dt} = 0.007 * \left(\frac{2}{3}\right) * 0.177 * 1.26 * 10^{-5} = 0.00104 * 10^{-5}$$

$$= 1.04 * 10^{-8} = 10.4 \frac{\text{nanograms}}{\text{yr}}$$