Assume otherwise. Then *R* has no nontrivial ideals, so it is a simple ring. Having only a finite number of left ideals, *R* must also be left artinian. Therefore, by Wedderburn's theorem, $R = M_n(D)$ for some division ring *D*, where, of course, $n \geq 2$. If $n \geq 3$, it is easy to produce (by column constructions) more than seven nontrivial left ideals in *R*. Thus, we must have $n = 2$. In this case, consider the map α : {lines through 0 in $(D^2)_D$} → {nontrivial left ideals in *R*}, defined by taking annihilators. Using linear algebra in the plane, we check readily that α is a 1–1 correspondence. Therefore, the number of nontrivial left ideals in *R* is $|D| + 1$. Thus, we have $|D| = 6$, which is impossible!

Two obvious examples of the rings *R* under consideration are the commutative rings $\mathbb{Z}4 \times \mathbb{Z}4$ and $\mathbb{Z}/2^8\mathbb{Z}$. For a noncommutative example, consider the ring $R = \begin{pmatrix} k & k \\ 0 & k \end{pmatrix}$, where *k* is a finite field. One computes that *R* has exactly $|k| + 3$ nontrivial left ideals, so taking $k = F_4$ gives what we want.