In the following, we assume $R = Mn(D)$, and identify R with End(VD) where dim$DV = n$. Note that, in this case, any nilpotent set $So \subseteq R$ will automatically satisfy $S_o^n = 0$. This follows readily by considering the chain of D-subspaces $V \supseteq S_oV \supseteq S_o^2V \supseteq \cdots$.
(For any subset $T \subseteq R$, TV denotes the D-subspace $\{ \sum_i t_i v_i : t_i \in T, v_i \in V \}$.)
Clearly, the set $S \subseteq R$ contains 0. Consider all nilpotent subsets $Si \subseteq S$ (e.g. $\{0, s\}$ for any $s \in S$). Since $S_i^n = 0$ for all i, Zorn's Lemma can be applied to show the existence of a *maximal* nilpotent subset $So \subseteq S$. We see easily that $\{0\} \subset So$. Let $U = SoV$. Then $0 \neq U \neq V$, so $\dim_D U$, $\dim_D V/U$ are both $< n$. Consider
$S1 := \{s \in S : sU \subseteq U\}$.
Clearly $S_1 \supseteq So$, and $S_1^2 \subseteq S_1$. Invoking an inductive hypothesis at this point, we may assume S_1 is nilpotent on U and on V/U. Then S_1 itself is nilpotent, and so $S_1 = So$. In particular, for any $s \in S \backslash So$, we have $sSo \not\subseteq So$ (for otherwise $sU = sSoV \subseteq SoV = U$ implies $s \in S_1 = So$).
Assume, for the moment, that $So \neq S$. Take $s \in S \backslash So$. Then $ss1 \notin So$ for some $s1 \in So$, and, since $ss1 \in S$, $(ss1)s2 \notin So$ for some $s2 \in So$, etc. But then we get $s(s1s2 \cdots sn) \notin So$ where all $si \in So$, contradicting the fact that $s(s1s2 \cdots sn) = 0 \in So$. Therefore, we must have $S = So$, and so $S^n = 0$.