

Consider a linear change of variables given by $x'_i = \sum_j c_{ij} x_j$ where $C = (c_{ij}) \in \text{GL}_m(k)$. We have $x'_i x'_j - x'_j x'_i = \sum_r c_{ir} x_r \sum_s c_{js} x_s - \sum_s c_{js} x_s \sum_r c_{ir} x_r = \sum_{r,s} c_{ir} c_{js} (x_r x_s - x_s x_r) = \sum_{r,s} c_{ir} a_{rs} c_{js}$. If we write $a'_{ij} = \sum_{r,s} c_{ir} a_{rs} c_{js}$ then $x'_i x'_j - x'_j x'_i = a'_{ij}$, and we have $A' = CAC^T$ where $A = (a_{ij})$ and $A' = (a'_{ij})$, and “ T ” denotes the transpose. Therefore, we are free to perform any congruence transformation on A . After a suitable congruence transformation, we may therefore assume that A consists of a number of diagonal blocks $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$, together with a zero block of size $t \geq 0$. If $t > 0$, then $\det(A) = 0$, and xm generates a proper ideal in R . If $t = 0$, then $\det(A) \neq 0$ and $m = 2n$ for some n . Here, R is the n th Weyl algebra $A_n(k)$. Since k has characteristic zero, R is a simple ring.