For any infinite cardinal $\beta \le \alpha$, let $E_\beta = \{f \in E : \text{rank}(f) < \beta\}$, where rank($f$) denotes the cardinal number $\dim_D f(V)$. Since $\text{rank}(g'fg) \le \text{rank}(f)$, E_β is an ideal of E. We claim that the ideals of E are (0), E and the E_β's. For this, we need the following crucial fact.

(*) *If $f, h \in E$ are such that* $\text{rank}(h) \le \text{rank}(f)$, *then* $h \in EfE$.

Indeed, write $V = \ker(h) \oplus V_1 = \ker(f) \oplus V_2$. Fix a basis $\{u_i : i \in I\}$ for V_1, and a basis $\{v_j : j \in J\}$ for V_2. We have $|I| = \text{rank}(h) \le \text{rank}(f) = |J|$, so let us assume, for convenience, that $I \subseteq J$. Define $g \in E$ such that $g(\ker(h)) = 0$, and $g(u_i) = v_i$ for all $i \in I$. Noting that $\{f(v_j) : j \in J\}$ are linearly independent, we can also find $g' \in E$ such that $g'(f(v_i)) = h(u_i)$ for all $i \in I$. Then $h = g'fg$. In fact, both sides are zero on ker(h), and on u_i ($i \in I$) we have $g'fg(u_i) = g'f(v_i) = h(u_i)$. This proves (*).

Now consider any ideal A <> 0,E. Then, for any $f \in$ A, $\text{rank}(f) < \alpha$. For, if $\text{rank}(f) = \alpha = \text{rank}(\text{Id})$, then (*) implies $\text{Id} \in EfE \subseteq$ A, a contradiction. Since the class of cardinal numbers is well-ordered, there exists a cardinal $\beta \le \alpha$ which is least among cardinals larger than rank(f) for every $f \in$ A. We leave to the reader the easy task of verifying that β is an infinite cardinal. Clearly, A $\subseteq E_\beta$ by the definition of E_β. We finish by showing that $E_\beta \subseteq$ A. Let $h \in E_\beta$, so $\text{rank}(h) < \beta$. By the choice of β, we must have $\text{rank}(h) \le \text{rank}(f)$ for some $f \in$ A. But then, by (*), $h \in EfE \subseteq$ A, as desired.