

Let  $D$  be a division ring,  $V = \bigoplus_{i=1}^{\infty} e_i D$ , and  $E = \text{End}(V_D)$ . Define  $f_1, f_2 \in E$  by  $f_1(e_n) = e_{2n}, f_2(e_n) = e_{2n-1}$  for  $n \geq 1$ .

We show that  $\{f_1, f_2\}$  form a free  $E$ -basis for  $E_E$ .

Define  $g_1, g_2 \in E$  by  $g_1(e_{2n-1}) = 0, g_1(e_{2n}) = e_n$ , and  $g_2(e_{2n-1}) = e_n, g_2(e_{2n}) = 0$  for  $n \geq 1$ . An easy calculation shows that  $f_1g_1 + f_2g_2 = 1 \in E$ , and  $g_2f_1 = g_1f_2 = 0$ . The former shows that  $\{f_1, f_2\}$  span  $E_E$ . To show that  $f_1, f_2$  are linearly independent in  $E_E$ , suppose  $f_1h_1 + f_2h_2 = 0$ , where  $h_i \in E$ . Then, for  $h := f_1h_1 = -f_2h_2$ , we have  $h = (f_1g_1 + f_2g_2)h = (f_1g_1)(-f_2h_2) + (f_2g_2)(f_1h_1) = 0$ . Since  $f_1, f_2$  are *injective* maps, it follows that  $h_1 = h_2 = 0$ . Therefore,  $E \sim E^2$  as right  $E$ -modules, and by induction,  $E \sim E^n$  for all  $n > 0$ .