

Let D be a division ring, $V = \bigoplus_{i=1}^{\infty} e_i D$, and $E = \text{End}(V_D)$. Define $f_1, f_2 \in E$ by $f_1(e_n) = e_{2n}, f_2(e_n) = e_{2n-1}$ for $n \geq 1$.

We show that $\{f_1, f_2\}$ form a free E -basis for E_E .

Define $g_1, g_2 \in E$ by $g_1(e_{2n-1}) = 0, g_1(e_{2n}) = e_n$, and $g_2(e_{2n-1}) = e_n, g_2(e_{2n}) = 0$ for $n \geq 1$. An easy calculation shows that $f_1 g_1 + f_2 g_2 = 1 \in E$, and $g_2 f_1 = g_1 f_2 = 0$. The former shows that $\{f_1, f_2\}$ span E_E . To show that f_1, f_2 are linearly independent in E_E , suppose $f_1 h_1 + f_2 h_2 = 0$, where $h_i \in E$. Then, for $h := f_1 h_1 = -f_2 h_2$, we have $h = (f_1 g_1 + f_2 g_2)h = (f_1 g_1)(-f_2 h_2) + (f_2 g_2)(f_1 h_1) = 0$. Since f_1, f_2 are *injective* maps, it follows that $h_1 = h_2 = 0$. Therefore, $E \sim E^2$ as right E -modules, and by induction, $E \sim E^n$ for all $n > 0$.