Question 1. *Given a sequence $\{a_n\}$ with a_n part of X and X is a metric space. Prove that if a_n is convergent, then the limit is unique.*

Solution. Let ρ denote the metric on X. Suppose there are $a, b \in X$ such that $a_n \to a$ and $a_n \to b$. Take an arbitrary $\varepsilon > 0$. By definition there are $N_1, N_2 \in \mathbb{N}$ such that $\rho(a_n, a) < \varepsilon$ for all $n > N_1$ and $\rho(a_n, b) < \varepsilon$ for all $n > N_2$. Choose $n > \max\{N_1, N_2\}$. Then by triangle inequality

$$\rho(a, b) < \rho(a, a_n) + \rho(a_n, b) < \varepsilon + \varepsilon = 2\varepsilon.$$

Since ε is an arbitrary positive number, we conclude that $\rho(a, b) = 0$. By identity of indiscernibles $a = b$. □

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