The answer is “no” in general. To construct an example, let R be the ring of 2 × 2 upper triangular matrices over a field k, and let M be the left R-module k^2 with the R-action given by matrix multiplication from the left. An easy computation shows that $E = \mathrm{End}(_RM) = k$, in particular, M is a semisimple E-module. We claim that M is not a semisimple R-module. In fact, consider the R-submodule
$N = \{(a,0)^t \mid a \in k\} \subseteq M$.

If $(b,c)^t$ is not in N, then c *is nonzero*, so $R\,(b,c)^t$ contains $\begin{pmatrix} 0 & ac^{-1} \\ 0 & 0 \end{pmatrix}\begin{pmatrix} b \\ c \end{pmatrix} = \begin{pmatrix} a \\ 0 \end{pmatrix}$ for all $a \in k$. This shows that N is not an R-direct summand of M, so M is not semisimple.