Conditions

for which value of k does the following pair of equation yield a unique solution for x such that the solution is positive x^2-y^2=0,(x-k)^2+y^2=1

Solution

$$\begin{cases} x^2 + y^2 = 0 \\ (x-k)^2 + y^2 = 1 \end{cases}$$

1st minus 2nd :

$$x^2 - (x-k)^2 + y^2 - y^2 = 0 - 1$$

$$x^2 - x^2 + 2kx - k^2 = -1$$

$$2kx - k^2 = -1$$

If k = 0, then $x \epsilon R$. If $k \neq 0$, then

$$x = \frac{k^2 - 1}{2k}$$

As x has a first power, this means that the solution is unique.

To find a positive solution for x, we must construct the inequality:

$$\frac{k^2 - 1}{2k} > 0$$

$$\frac{(k-1)(k+1)}{2k} > 0$$

It's obvious, that the $k \epsilon (-1,0) \cup (1,\infty)$

Answer: $\boldsymbol{k \epsilon (-1,0) \cup (1,\infty)}$