Solve $y'' - 5y' - 6y = e^{-2t}$, where $y'(0) = 1$ and $y(0) = -1$ using Laplace Transform.

Solution:

$$y \to Y(p)$$

$$y' \to pY(p) - y(0) = pY(p) + 1$$

$$y'' \to p^2Y(p) - py(0) - y'(0) = p^2Y(p) + p - 1$$

$$e^{-2t} \to \frac{1}{p+2}$$

$$p^2Y(p) + p - 1 - 5(pY(p) + 1) - 6Y(p) = \frac{1}{p+2} \to (p^2 - 5p - 6)Y(p) = \frac{1}{p+2} - p + 6$$

$$(p^2 - 5p - 6)Y(p) = \frac{-p^2 + 4p + 13}{p+2} \to Y(p) = \frac{-p^2 + 4p + 13}{(p+2)(p^2 - 5p - 6)} = \frac{-p^2 + 4p + 13}{(p+2)(p+1)(p-6)}$$

$$Y(p) = \frac{A}{(p+2)} + \frac{B}{(p+1)} + \frac{C}{(p-6)} = \frac{(A+B+C)p^2 + (-5A - 4B + 3C)p + (-6A - 12B + 2C)}{(p+2)(p+1)(p-6)}$$

$$\begin{cases} A + B + C = -1 \\ -5A - 4B + 3C = 4 \\ -6A - 12B + 2C = 13 \end{cases} \to \begin{cases} A = \frac{1}{8} \\ B = -\frac{8}{7} \\ C = \frac{1}{56} \end{cases}$$

$$Y(p) = \frac{1}{8}\frac{1}{(p+2)} - \frac{8}{7}\frac{1}{(p+1)} + \frac{1}{56}\frac{1}{(p-6)} \leftarrow \frac{1}{8}e^{-2t} - \frac{8}{7}e^{-t} + \frac{1}{56}e^{6t} = y(t)$$

Answer: $y(t) = \frac{1}{8}e^{-2t} - \frac{8}{7}e^{-t} + \frac{1}{56}e^{6t}$.