

Question #16952 suppose that you own 8 math books and 6 computer science books and wish to fill 7 position on a shelf. what is the probability that the first 4 position are to be occupied by math books and the last 3 by computer science books?

Solution. First compute the total number of filling the 7 position by 8 math books and 6 computer science. First, suppose that we take only math books, then we will get $7! \binom{8}{7}$, suppose that we take k books on computer science $k = \overline{1, 6}$, then we will get $\binom{7}{k} \binom{6}{k} k! \cdot \binom{8}{7-k} (7-k)!$ variants of filling. To sum it up, the total number of filling possibilities is $7! \binom{8}{7} + \sum_{k=1}^6 \binom{7}{k} \binom{6}{k} k! \cdot \binom{8}{7-k} (7-k)!$. The number of elementary events that promote “the first 4 position are to be occupied by math books and the last 3 by computer science books” equals $\binom{8}{4} 4! \cdot \binom{6}{3} 3!$, thus the probability equals

$$\frac{\binom{8}{4} 4! \cdot \binom{6}{3} 3!}{7! \binom{8}{7} + \sum_{k=1}^6 \binom{7}{k} \binom{6}{k} k! \cdot \binom{8}{7-k} (7-k)!}$$

Answer. $\frac{\binom{8}{4} 4! \cdot \binom{6}{3} 3!}{7! \binom{8}{7} + \sum_{k=1}^6 \binom{7}{k} \binom{6}{k} k! \cdot \binom{8}{7-k} (7-k)!}$