

If one of  $U, V$  is finitely generated, the answer is “yes”. In general, however, the answer is “no”, as we can show by the following example over  $R = \mathbb{Z}$ . Take  $U = V = \bigoplus_p \mathbb{Z}_p$ , where  $p$  ranges over all primes. Let  $\varepsilon \in E := \text{Hom}_{\mathbb{Z}}(V, V)$  be the identity map from  $V$  to  $V$ . We claim that  $\varepsilon$  has *infinite* additive order in  $E$  (which certainly implies that  $E$  is not a semisimple  $\mathbb{Z}$ -module). For any natural number  $n$ , take a prime  $p > n$ . Then

$$(n \cdot \varepsilon)(0, \dots, 1, 0, \dots) = (0, \dots, n, 0, \dots) \neq 0$$

if the 1 appears in the coordinate corresponding to  $\mathbb{Z}_p$ . Therefore,  $n \cdot \varepsilon \neq 0 \in E$ , as claimed.