

If one of U, V is finitely generated, the answer is “yes”. In general, however, the answer is “no”, as we can show by the following example over $R = \mathbb{Z}$. Take $U = V = \bigoplus_p \mathbb{Z}_p$, where p ranges over all primes. Let $\varepsilon \in E := \text{Hom}_{\mathbb{Z}}(V, V)$ be the identity map from V to V . We claim that ε has *infinite* additive order in E (which certainly implies that E is not a semisimple \mathbb{Z} -module). For any natural number n , take a prime $p > n$. Then $(n \cdot \varepsilon)(0, \dots, 1, 0, \dots) = (0, \dots, n, 0, \dots) \neq 0$ if the 1 appears in the coordinate corresponding to \mathbb{Z}_p . Therefore, $n \cdot \varepsilon \neq 0 \in E$, as claimed.