

(1) \Rightarrow (4). Let $M = \bigoplus_{i \in I} M_i$ where the M_i 's are simple modules. If M is generated by m_1, \dots, m_n , we have $\{m_1, \dots, m_n\} \subseteq \bigoplus_{i \in J} M_i$ for a finite subset $J \subseteq I$. Therefore, $M = \bigoplus_{i \in J} M_i$ (which of course implies that $J = I$). (4) \Rightarrow (2) \Rightarrow (1) and (4) \Rightarrow (3) are trivial, so we are done if we can show (3) \Rightarrow (4). Let $M = \bigoplus_{i \in I} M_i$ as above. If I is infinite, this decomposition of M would lead to a strictly descending chain of submodules of M . Therefore, I must be finite, and we have (4).