

First suppose R is semisimple. It is obvious that I must be finite. Conversely, assume I is finite, and consider any ideal $A \subseteq R$. Then, $A = \bigoplus_{i \in J} F_i$ for a subset $J \subseteq I$. Clearly, A is a direct summand of ${}_R R$, so ${}_R R$ is a semisimple module, as desired.