The answer is “no”: for instance, $A = R_1 \times R_2 \times \cdots$ (where R_i are nonzero rings) cannot be embedded as a subring of a (left) semisimple ring. Indeed, if A is a subring of a ring R, then R will have nonzero idempotents e_1, $e_2, \ldots$ with $e_i * e_j = 0$ for $i <> j$. But then $R \supseteq Re_1 \oplus Re_2 \oplus \cdots$, and this implies that R is not left noetherian (let alone left semisimple).

Similarly, if k is any nonzero ring, $A = k[x_1, x_2, \ldots]$ with the relations $x_i * x_j = 0$ (for all i, j) cannot be embedded in a left semisimple ring. Indeed, if A is a subring of a ring R, then $Rx_1 \subset Rx_1 + Rx_2 \subset \cdots$ (by an easy proof), and again R is not left noetherian (let alone left semisimple).