Define a ring homomorphism $\varphi : B \to M_2(k[t])$ by $\varphi \mid k = \mathrm{Id}_k$, and

$$\varphi(x) = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \ \varphi(y) = \begin{pmatrix} 0 & 1 \\ t & 0 \end{pmatrix}.$$

(It is easy to check that φ respects the relations $x^2 = 0$ and $xy + yx = 1$ on B.) We have $\varphi(y^2) = t \cdot I_2$, so

$(*)$ $\varphi(y^{2n}) = t^n \cdot I_2$, and $\varphi(y^{2n+1}) = \begin{pmatrix} 0 & t^n \\ t^{n+1} & 0 \end{pmatrix}$. Expressing B in the form $k[y] + k[y]x$, we can write an arbitrary element $\gamma \in B$ in the form $\alpha + \beta x$, where $\alpha = \Sigma\, a_i y_i$, and $\beta = \Sigma\, b_i y_i$ (with $a_i, b_i \in k$). In view of $(*)$,

$$\varphi(\alpha) = \begin{pmatrix} a_0 + a_2 t + a_4 t^2 + \ldots & a_1 + a_3 t + a_5 t^2 + \ldots \\ a_1 t + a_3 t^2 + a_5 t^3 + \ldots & a_0 + a_2 t + a_4 t^2 + \ldots \end{pmatrix}$$

$$\varphi(\beta x) = \begin{pmatrix} b_1 t + b_3 t^2 + b_5 t^3 + \ldots & 0 \\ b_0 + b_2 t + b_4 t^2 + \ldots & 0 \end{pmatrix}.$$

If $\varphi(\gamma) = 0$, we must have all $a_i = 0$, and therefore all $b_i = 0$. This shows φ is one-one. The form of the matrices above also shows that φ is onto, so $B \sim M_2(k[t])$.