

We induct on n . First assume $n = 1$ and write $A = R[x]$. Let $g \in A \setminus \{0\}$ be of minimal degree such that $I \cdot g = 0$, say $g = bx^d + \dots, b \neq 0$. If $d > 0$, then $f \cdot b \neq 0$ for some $f = \sum_i a_i x^i \in I$. We must have $a_i g \neq 0$ for some i (for otherwise $a_i b = 0$ for all i and hence $f \cdot b = 0$). Now pick j to be the largest integer such that $a_j g \neq 0$. From $0 = f \cdot g = (a_j x^j + \dots + a_0) \cdot g$, we have $a_j b = 0$, so $\deg(a_j g) < d$. But $I \cdot (a_j g) \subseteq I \cdot g = 0$, which contradicts the choice of g . Thus, we must have $d = 0$ and $g \in R$. For the inductive step, write $A = B[x_n]$, where $B = R[x_1, \dots, x_{n-1}]$. Each polynomial $f \in A$ can be written in the form $\sum_i h_i x_n^i$ ($h_i \in B$). For convenience, let us call the h_i 's the “coefficients” of f . Assume $I \cdot g = 0$, where $g \neq 0$ in A . By the paragraph above, we may assume that $g \in B$. Let I' be the right ideal in B generated by all “coefficients” of the polynomials in I . Since g does not involve x^n (and I is a right ideal), we see easily that $I \cdot g = 0 \Rightarrow I' \cdot g = 0$. By the inductive hypothesis, there exists $r \in R \setminus \{0\}$ such that $I' \cdot r = 0$. From this, we have, of course, $I \cdot r = 0$, as desired.