

(1) Let  $R = k\langle x, y \rangle$  and  $A = xR$ . For  $r \in R$ , we have  $r \in \text{IR}(A)$  iff  $r \cdot x \in xR$ . Writing  $r = r_0 + r_1$  where  $r_0$  is the constant term of  $r$ , we see that  $r \cdot x = r_0x + r_1x \in xR$  iff  $r_1 \in xR$ .

This shows that  $\text{IR}(A) = k + xR$ , from which we get  $\text{ER}(A) = (k + xR)/xR \sim k$ .

(2) Let  $R = \mathbb{Z} \oplus \mathbb{Z}i \oplus \mathbb{Z}j \oplus \mathbb{Z}k$  and  $A = xR$ , where  $x = i + j + k$ . Since  $x^2 = -3$ , we have  $3R \subseteq xR$ . Writing “bar” for the projection map  $R \rightarrow \bar{R} = R/3R$ , we check easily that the right annihilator of  $x$  in  $R$  has 9 elements. Since  $|\bar{R}| = 3^4$ , it follows that

$$(*) [R : xR] = [\bar{R} : x\bar{R}] = 81/9 = 9.$$

Now  $xR$  is not an ideal in  $R$ , so we have  $R \supset \text{IR}(xR) \supseteq xR + \mathbb{Z} \supset xR$ .

From (\*), we see that  $\text{IR}(xR) = xR + \mathbb{Z}$ , and  $\text{ER}(xR) = \text{IR}(xR)/xR \sim \mathbb{Z}/3\mathbb{Z}$ .