Write a matrix $\beta \in R$ in the block form $\begin{pmatrix} x & y \\ z & w \end{pmatrix}$, where $x \in \mathrm{M}r(k)$, and similarly, write $\alpha \in A$ in the block form $\alpha = \begin{pmatrix} 0 & 0 \\ u & v \end{pmatrix}$. Since $\beta\alpha = \begin{pmatrix} x & y \\ z & w \end{pmatrix}\begin{pmatrix} 0 & 0 \\ u & v \end{pmatrix} = \begin{pmatrix} yu & yv \\ wu & wv \end{pmatrix}$, the condition for $\beta A \subseteq A$ amounts to $y = 0$. Therefore, I$R(A)$ is given by

the ring of "block lower-triangular" matrices $\left\{\begin{pmatrix} x & 0 \\ z & w \end{pmatrix}\right\}$. Quotienting out the ideal $\left\{\begin{pmatrix} 0 & 0 \\ z & w \end{pmatrix}\right\}$, we get the eigenring E$R(A) \sim \mathrm{M}r(k)$.