

Define $\lambda : IR(A) \rightarrow \text{End}R(R/A)$ by taking $\lambda(r)$ ($r \in IR(A)$) to be left multiplication by r . (Since $rA \subseteq A$, $\lambda(r)$ is a well-defined endomorphism of the module $(R/A)R$.) Now $\lambda(r)$ is the zero endomorphism iff $rR \subseteq A$, that is, $r \in A$. Since λ is a ring homomorphism, it induces a ring embedding $IR(A) \rightarrow \text{End}R(R/A)$. We finish by showing that this map is *onto*. Given $\varphi \in \text{End}R(R/A)$, write $\varphi(\bar{1}) = \bar{r}$, where $r \in R$. Then $\varphi(\bar{x}) = \varphi(\bar{1} \cdot x) = \bar{r} \cdot x = \overline{rx}$ for any $x \in R$. In particular, for $x \in A$, we see that $rx \in A$, so $r \in IR(A)$, and we have $\varphi = \lambda(r)$.
