Notice the fact: $m \in \mathbb{Z}$ is a square in R iff m is a square in $\mathbb{Z}/n\mathbb{Z}$.
Applying this with $n = m = 2p$, we see that $2p \in R^2$. Next, we apply again fact with $n = 2p$ and $m = p$. Since $p2 - p = p(p - 1) \in 2p\mathbb{Z}$, p is a square in $\mathbb{Z}/2p\mathbb{Z}$, so $p \in R^2$. Finally, we apply again fact with $n = 2p$ and $m = 2$. By above fact, $2 \in R^2$ iff 2is a square in $\mathbb{Z}/2p\mathbb{Z}$. Since $\mathbb{Z}/2p\mathbb{Z} \cong \mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/p\mathbb{Z}$, this holds iff 2is a square in $\mathbb{Z}/p\mathbb{Z}$.