

Let $A = 2\mathbb{Z}_{2^k} \subset \mathbb{Z}_{2^k}$. Then $\begin{pmatrix} A & \mathbb{Z}_2 \\ 0 & 0 \end{pmatrix} \subset N$, since any element in the former has k th power equal to zero.

Conversely, if $\begin{pmatrix} m & a \\ 0 & n \end{pmatrix} \in N$, then $m \in \mathbb{Z}_{2^k}$ and $n \in \mathbb{Z}_2$ must both be nilpotent, so $m \in A$ and $n = 0$. This shows

that $N = \begin{pmatrix} A & \mathbb{Z}_2 \\ 0 & 0 \end{pmatrix}$.