

Let  $A = 2\mathbb{Z}_{2^k} \subset \mathbb{Z}_{2^k}$ . Then  $\begin{pmatrix} A & \mathbb{Z}_2 \\ 0 & 0 \end{pmatrix} \subset N$ , since any element in the former has  $k$ th power equal to zero.

Conversely, if  $\begin{pmatrix} m & a \\ 0 & n \end{pmatrix} \in N$ , then  $m \in \mathbb{Z}_{2^k}$  and  $n \in \mathbb{Z}_2$  must both be nilpotent, so  $m \in A$  and  $n = 0$ . This shows that  $N = \begin{pmatrix} A & \mathbb{Z}_2 \\ 0 & 0 \end{pmatrix}$ .