One can also show that $R \sim R^{op} \Rightarrow k \sim k^{op}$. Then since R is isomorphic to S (under mentioned assumptions), and k has an anti-automorphism, and the same is true for A, R and S, then statement is obvious.
In details:
To simplify the notations, we shall work in the (sufficiently typical) case $n = 3$. Suppose $\varepsilon : k \to k$ is an anti-automorphism (resp. involution). Composing the transpose map with ε on matrix entries, we can define $\delta_0 : A \to A$ with

$$\delta_0 \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} = \begin{pmatrix} \varepsilon(a) & \varepsilon(d) & \varepsilon(g) \\ \varepsilon(b) & \varepsilon(e) & \varepsilon(h) \\ \varepsilon(c) & \varepsilon(f) & \varepsilon(i) \end{pmatrix}$$

It is easy to check that this δ_0 is an anti-automorphism (resp. involution) of A, and therefore so is $\delta := \alpha \circ \delta_0$ given by

$$\delta \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} = \begin{pmatrix} \varepsilon(i) & \varepsilon(f) & \varepsilon(c) \\ \varepsilon(h) & \varepsilon(e) & \varepsilon(b) \\ \varepsilon(g) & \varepsilon(d) & \varepsilon(a) \end{pmatrix}$$

Thus R, S, R^op, S^op are all isomorphic.